

Lesson 2-2: Biconditionals & Definitions

Playing with conditionals some more

Yesterday we learned about conditional statements. We messed around with them and discovered converse statements. Today we are going to see what else we can come up with.

Looking at converse statements we discovered that just because the conditional they came from is true, it doesn't mean the converse is true. Some are and some aren't. When we do find a conditional and converse pair that is both true, we have statements that work in both directions: $p \rightarrow q$ and $q \rightarrow p$. We call this a **biconditional**.

Biconditionals

We have a compact way of writing biconditionals by joining the halves of the conditional and converse statement with the phrase *if and only if*.

Take a look at the *Check Understanding #1* on page 75. It asks us to write the converse of the conditional. If the converse is true, then combine the two statements in a biconditional.

Conditional: If three points are collinear, then they lie on the same line. (True)

Converse: If three points lie on the same line, then they are collinear. (True)

Biconditional: Three points are collinear if and only if they lie on the same line.

Many times we will shorten the phrase "if and only if" to "iff". The above biconditional could then be phrased:

Three points are collinear iff they lie on the same line.

And ... to take it another step ... we can write the biconditional in symbol form too:

$$p \leftrightarrow q$$

meaning statement p if and only if statement q .

Using this symbol form, we can define biconditional a bit more formally:

A biconditional combines $p \rightarrow q$ and $q \rightarrow p$ as $p \leftrightarrow q$.

Taking a biconditional apart

You can go the other way too ... you can break a biconditional into its parts: the conditional statement and its converse:

1. Use the left side of the *iff* as the hypothesis
2. Use the right side of the *iff* as the conclusion
3. Form the conditional statement
4. Create the converse of this conditional statement

Lesson 2-2: Biconditionals & Definitions

Consider problem #8 on page 78:

Biconditional: An integer is divisible by 100 if and only if its last two digits are zeros.

Conditional: If an integer is divisible by 100, then its last two digits are zeros.

Converse: If an integer's last two digits are zeros, then it is divisible by 100.

What makes a definition a good definition?

You have been getting definition after definition thrown at you. It would be valid to wonder how to tell if a definition is good or not. Well by now, I'm sure you won't be surprised that in math, we have laid out rules for what is a good definition:

A good definition:

1. Uses clearly understood terms...
 - Use only terms that are commonly understood or already defined.
2. Is precise...
 - Avoid words such as *large*, *sort of*, *some*, *part*, etc.
3. Is reversible...
 - In other words ... it is a biconditional!
 - In fact, one way to show that a definition is not a good definition is to find a counter-example.

Let's practice this a bit. Take a look at *Check Understanding #3* on page 77. Show that this definition of a *right angle* is reversible. Then write it as a true biconditional.

Definition: A right angle is an angle whose measure is 90.

Conditional: If an angle is a right angle, then its measure is 90. (True)

Converse: If the measure of an angle is 90, then it is a right angle. (True)

Biconditional: An angle is a right angle iff its measure is 90.

Consider *Check Understanding #4* on page 77. Is the following statement a good definition? Explain.

Definition: A square is a figure with four right angles.

Does it use clearly understood terms? Yes

Is it precise? Yes

Is it reversible?

Conditional: If a figure is a square, then it has four right angles. (True)

Converse: If a figure has four right angles, then it is a square. (False)

Counter-example: A rectangle has four right angles but is not a square.

Not reversible

Not a good definition.

Lesson 2-2: Biconditionals & Definitions

Now consider problem #18 on page 78. Is this statement a good definition? If not, explain:

Proposed definition: A cat is an animal with whiskers.

Does it use clearly understood terms? Yes

Is it precise? Yes

Is it reversible?

Conditional: If the animal is a cat, then it has whiskers. (True)

Converse: If the animal has whiskers, then it is a cat (False)

Counter-example: A mouse has whiskers but is not a cat.

Not reversible.

Not a good definition

Look at problem #20 on page 78. Is this statement a good definition? If not, explain:

Proposed definition: A segment is part of a line.

Does it use clearly understood terms? Yes

Is it precise? No

“part of a line” is ambiguous

Both a ray and point are parts of a line.

Not precise

Not a good definition

Assign homework

p. 78 1-13 odd, 17-23 odd, 27-35 odd, 41, 43-46